King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 253: Discrete Structures I
Summer Semester 2016-2017
Final Exam, Monday August 21, 2017.

Name:

ID#:

Instructions:

1. This exam consists of **nine** pages, including this page and the final reference sheet, containing **six** questions.
2. You have to answer all **six** questions.
3. The exam is closed book and closed notes. Non-programmable calculators are allowed. Make sure you turn off your mobile phone and keep it in your pocket if you have one.
4. The questions are **not equally weighed**.
5. This exam is out of **100** points.
6. You have exactly **120** minutes to finish the exam.
7. Make sure your answers are **readable**.
8. If there is no space on the front of the page, feel free to use the back of the page. Make sure you indicate this in order for me not to miss grading it.

Question Number	Maximum # of Points	Earned Points
I	**20**	
II	**15**	
III	**15**	
IV	**15**	
V	**20**	
VI	**15**	
Total	**100**	

I. (20 points) Choose the correct answer from the following choices.

1. The inverse of "it rains today only if I drive to work" is
   - (a) if I don't drive to work, then it does not rain today.
   - (b) if it does not rain today, then I don't drive to work.
   - (c) if I drive to work, then it rains today.
   - (d) if it rains today, then I drive to work.
   - (e) none of the above.

2. On the island of knights and knaves you encounter two people. A and B. Person A says, "B is a knave." Person B says, "At least one of us is a knight." Determine whether each person is a knight or a knave.
   - (a) B is a knave, but A cannot be determined.
   - (b) A is a knave and B is a knight.
   - (c) A is a knight and B is a knight.
   - (d) A is a knave and B is a knave.
   - (e) A is a knave and B cannot be determined.

3. Let A and B be two finite sets with $|A| = 200$, $|B| = 250$ and $|A \cap B| = 50$. Then, $|A \oplus B| =$
   - (a) 500
   - (b) 450
   - (c) 400
   - (d) 350
   - (e) 300

In Questions 4-5, let $A(y)$: y is an advanced course, $F(x)$: x is a freshman, $T(x, y)$: x is taking y and $P(x, y)$: x passed y, where variable x represents students and the variable y represents courses.

4. The statement "Every freshman taking an advanced course has passed it" is represented by
   - (a) $\forall x \forall y \big(F(x) \wedge A(y) \wedge T(x, y) \wedge P(x, y)\big)$.
   - (b) $\forall x \forall y \Big(F(x) \wedge \big(\big(T(x, y) \wedge A(y)\big) \rightarrow P(x, y)\big)\Big)$.
   - (c) $\forall x \forall y \Big(\big(F(x) \rightarrow A(y) \wedge T(x, y)\big) \wedge P(x, y)\Big)$.
   - (d) $\forall x \forall y \Big(\big(F(x) \wedge A(y)\big) \rightarrow T(x, y) \rightarrow P(x, y)\Big)$.
   - (e) $\forall x \forall y \Big(\big(F(x) \wedge A(y) \wedge T(x, y)\big) \rightarrow P(x, y)\Big)$.

5. The statement "No one is taking every advanced course" is represented by
   - (a) $\forall y \exists x \big(A(y) \rightarrow \neg T(x, y)\big)$.
   - (b) $\forall y \exists x \big(A(y) \wedge \neg T(x, y)\big)$.
   - (c) $\forall x \exists y \big(A(y) \rightarrow \neg T(x, y)\big)$.
   - (d) $\forall x \exists y \big(A(y) \wedge \neg T(x, y)\big)$.
   - (e) none of the above.

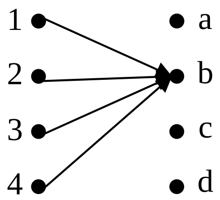
6. $A - (B \cap C)$ is equivalent to
   - (a) $(A - C) \cup (A - B)$.
   - (b) $(A - B) \cap (A - C)$.
   - (c) $(A - B) \cup$ C.
   - (d) $A \cup (B - C)$.
   - (e) more than one answer above.

7. $\bigcap_{i=1}^{\infty}\left(-\frac{1}{i}, 1-\frac{1}{i}\right)=$
   - (a) (0,1).
   - (b) [0,1].
   - (c) {0}.
   - (d) {1}.
   - (e) Φ.

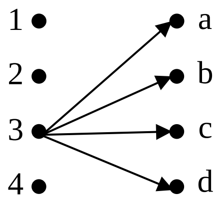
8. The amounts of postage that can be formed using just 5-cent and 6-cent stamps include all positive integer values n where
   - (a) $n \geq 10$.
   - (b) $n \geq 11$.
   - (c) $n \geq 15$.
   - (d) $n \geq 18$.
   - (e) $n \geq 20$.

9. How many positive integers between 100 and 1000 inclusive are divisible by 3 but not 5?
   - (a) 120
   - (b) 240
   - (c) 267
   - (d) 300
   - (e) None of the above.

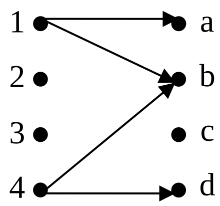
10. Which of the following represents a partial function, but not a total function?



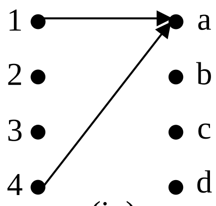
(i)



(ii)



(iii)



(iv)

   - (a) (iv) only.
   - (b) (iii) and (iv).
   - (c) (ii), (iii) and (iv).
   - (d) (i) only.
   - (e) None.

II. (15 points) Answer the following questions.

1. (8 points) Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, and so on. [*Hint:* For the inductive step, separately consider the case where $k + 1$ is even and where it is odd.]

2. (2 points) In your answer to part 1, justify the number of cases you considered for the basis step.

3. (5 points) Give a recursive definition of the set
$$S = \{(a, b) | a \in Z^+, b \in Z^+, and\ 3 | a + b\}$$
of ordered pairs of positive integers. [*Hint:* Plot the points in the set in the plane and look for lines containing points in the set.]

III. (15 points) In all questions below, make sure that you clearly justify your answer.

1. (5 points) The name of a variable in the JAVA programming language is a string of between 1 and 65,535 characters, inclusive, where each character can be an uppercase or a lowercase letter, a dollar sign, an underscore, or a digit, except that the first character must not be a digit. Determine the number of different variable names in JAVA.

$$\sum_{j=0}^{65534} (26 + 26 + 1 + 1) * (26 + 26 + 1 + 1 + 10)^j$$

2. (10 points) Suppose that a general purpose lab has 100 computers and 20 printers. A cable can be used to directly connect a computer to a printer. For each printer, only one direct connection to that printer can be active at any time.
   (a) (3 points) What is the maximum number of cables that can connect the 100 computers to the 20 printers?

   100*20=2000

   (b) (7 points) Find the least number of cables required to connect the 100 computers to the 20 printers to guarantee that every subset of 20 computers can directly access 20 different printers. Justify your answer.

20+80*20=1620

if we have 1619 then there is at least one printer connected to 80 computers, then the remaining 19 printers must serve 20 computers which is impossible

IV. (15 points) In all questions below, make sure that you clearly justify your answer.

1. (10 points) How many license plates consisting of three letters followed by one to four digits contain no letter or digit twice?

26*25*24*10(9*8*7+9*8+9+1)

2. (5 points) Find the value of

$$\sum_{j=1}^{1000} \binom{1000}{j} (-2)^j$$

$= (1-2)^{1000} - (1-2)^{0} = 1-1 = 0$

V. (20 points) In all questions below, make sure that you clearly justify your answer.

1. (5 points) What is the probability that a five-card poker hand contains exactly one ace?

   (C(4,1)*C(48,4))/C(52,5)

2. (5 points) Find the probability of selecting none of the correct six integers in a lottery, where the order in which these integers are selected does not matter, from the positive integers not exceeding 56.

   C(50,6)/C(56,6)

3. (10 points) In roulette, a wheel with 40 numbers is spun. Of these, 19 are red, and 19 are black. The other two numbers, which are neither black nor red, are 0 and 00. The probability that when the wheel is spun it lands on any particular number is 1/40.

   (a) What is the probability that the wheel lands on a red number?

   19/40

   (b) What is the probability that the wheel lands on a black number twice in a row?

   $(19/40)^2$

   (c) What is the probability that the wheel lands on 0 or 00?

   2/40

   (d) What is the probability that in five spins the wheel never lands on either 0 or 00?

   $(38/40)^5$

VI. (15 points) In all questions below, make sure that you clearly justify your answer.

1. (7 points) A string that contains only 0s, 1s, and 2s is called a ternary string.
   (a) Find a recurrence relation for the number of ternary strings of length n that contain three consecutive 0s.

$$a_n = 2*(a_{n-1}+a_{n-2}+a_{n-3})+3^{n-3}$$

   (b) What are the initial conditions?

a1 = 0
a2 = 0
a3 = 1

2. (8 points) Solve the following recurrence relation with together with the initial conditions given:

$$a_n = 4a_{n-1} - 4a_{n-2} \text{ for } n \geq 2, a_0 = 6, a_1 = 8.$$

$x^2 -4x+4 = 0 \text{ -> } (x-2)^2 = 0$
$x = 2$
$a_n = 2^n(\alpha n+\beta)$
$6 = \beta$
$8 = 2(\alpha+6)$
$4 = \alpha+6$
$\alpha = -2$
$an = 2^n(6-2n)$

Some Useful Formulas

$\mathbb{N} = \{0,1,2,3,\dots\}$ $\quad \mathbb{Z} = \{\dots,-2,-1,0,1,2,\dots\}$ $\quad \mathbb{Q} =$ set of rational numbers
$\mathbb{R} =$ set of real numbers

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad , \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad , \quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1} \text{ where } a \neq 1 \quad , \quad \sum_{i=0}^{\infty} a^i = \frac{1}{1-a} \text{ where } |a| < 1,$$

$$\sum_{i=0}^{n} ic^i = \sum_{i=1}^{n} ic^i = \frac{nc^{n+2} - nc^{n+1} - c^{n+1} + c}{(c-1)^2}$$

$$\sum_{i=1}^{\infty} ia^{i-1} = \frac{1}{(1-a)^2} \text{ where } |a| < 1$$

$p \to (p \vee q)$	Addition	$[\neg q \wedge (p \to q)] \to \neg p$	Modus Tollens
$(p \wedge q) \to p$	Simplification	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$((p) \wedge (q)) \to (p \wedge q)$	Conjunction	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
$[p \wedge (p \to q)] \to q$	Modus Ponens	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution

Some Useful Sequences	
n^2	1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, …
n^3	1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, …
n^4	1, 16, 81, 256, 625, 1296, 2401, 4096, 6561, 10000, 14641, …
2^n	2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, …
3^n	3, 9, 27, 81, 243, 729, 2187, 6561, 19683, 59049, 177147, …
$n!$	1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, 39916800
f_n Fibonacci	1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, …

$A \cap U = A$ $A \cup \Phi = A$	Identity Laws	$A \cup U = U$ $A \cap \Phi = \Phi$	Domination Laws
$A \cap A = A$ $A \cup A = A$	Idempotent Laws	$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative Laws
$\overline{(\bar{A})} = A$	Complementation Law	$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative Laws
$\overline{A \cap B} = \bar{A} \cup \bar{B}$ $\overline{A \cup B} = \bar{A} \cap \bar{B}$	De Morgan's Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive Laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption Laws	$A \cup \bar{A} = U$ $A \cap \bar{A} = \Phi$	Complement Laws